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Who should *not* share? The merits of withholding unused vehicles

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Abstract

People repeatedly demand travel, using available vehicles scattered around space. What can justify vehicle withholding (i.e. preventing others from using it, for own future use) from the social welfare perspective? This paper investigates heterogeneity in the potential cost of search for alternative vehicles as such justification. It is shown that travellers whose search cost is substantially higher than that of others (e.g. limited-mobility people) can optimally withhold a vehicle. The heterogeneity of search costs should be sufficiently strong, e.g. a uniform distribution is not variable enough to justify withholding by anyone. In an example calibrated for car use in London, it is shown that at most 39% of car users should withhold their vehicles under the most extreme modelling assumptions, while all others should share.

Keywords: Vehicle sharing, Transportation demand, Spatial search frictions

JEL codes: D61, L92, O18, R40

1. Introduction

Vehicle sharing is a technology that allows the same vehicle to be used by different people at different points in time, reducing the number of vehicles that are needed to meet transportation demand. Today, shared vehicles (SV) remain a fringe transportation option and serve only a fraction of travel demand, while exclusive-use vehicles continue to be viewed by most people as the default option. This paper turns the comparison on its head and

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assumes vehicle sharing is the default option, while vehicle *withholding* is an alternative option. Here withholding is defined as prevention of other people from legally using the vehicle, for the purpose of own future use. This paper asks the question: what can justify, from the social welfare perspective, such withholding by some people?

Historically, exclusive nature of vehicle use was driven by lack of technology for effective sharing. For example although carsharing was first attempted in 1948 (Shaheen et al., 2001), it has reached commercial success only in the 21st century, after new communication technologies allowed to remotely grant control of the vehicle to a specific individual. Today, the technological challenge of vehicle sharing is successfully solved with minimal additional equipment required for the vehicle.

Another potential motivation for vehicle exclusive access is the moral hazard problem: users may show more care about the vehicle if they expect they will continue to use the same vehicle in the future. Modern carsharing companies, however, are increasingly able to track vehicle movements and detect the driving style, reducing the moral hazard problem. The rest of this paper does not consider this problem.

The rest of the paper is focused on analysing whether vehicle withholding by some people travelling (*travellers* henceforth) can be justified by heterogeneity of such people. The paper is focused on one dimension of such heterogeneity, while other potential dimensions are mentioned in the conclusion.

While vehicle sharing allows to meet the same travel demand with fewer vehicles, it also creates spatial search frictions. As available vehicles are scattered around space, they have to be searched (usually via a mobile app) and then the traveller normally has to walk to the vehicle of choice.

This paper explores whether traveller heterogeneity in walking costs (i.e. in costs of physical effort or in the opportunity cost of time) can justify vehicle withholding by some travellers. The answer is yes, but only if the walking cost heterogeneity is sufficiently high, and only for a minority of travellers with the highest walking cost. In particular, it is shown that a uniform distribution of walking costs is not variable enough and does not justify withholding by anyone. In an automobile travel demand example calibrated for the city of London, the maximum share of travellers who should optimally withhold their vehicles is 39% in the most adverse scenario, and is much lower than that under more plausible scenarios. All remaining travellers should optimally share automobiles. The result is proven for arbitrary distribution of walking costs with a calibrated upper bound.

It should be emphasised that it is not the absolute level of walking cost that matters for optimal withholding, but its comparison with that of other travellers. When the walking cost is homogenous, regardless of its level, the first-come-first-serve basis for vehicle use maximises welfare, so withholding is not justified. Withholding by traveler A can be optimal only if other travellers typically have much lower walking costs, so the cost of withholding (i.e. increased search cost for other travellers, and longer period of inactivity of the withheld vehicle) is below its benefit (allowing traveller A to avoid walking in the future).

This paper contributes to my own stream of literature on various economic-theoretical aspects of vehicle sharing. In Zakharenko (2023a), I develop a spatially explicit model of transportation demand, derive a formula for location-specific pricing for SV services, and predict how this formula can improve the existing practice of carsharing. In Zakharenko (2023b), I provide a theoretical argument for free municipal parking for shared vehicles, as a method to achieve a "big push" from private to shared mobility. A number of papers model search frictions in taxi or ridehailing, e.g. Frechette et al. (2019) and Buchholz (2022), or in a wider context of transportation demand (Brancaccio et al., 2023). All of these models however assume that the agents demanding travel are short-lived, i.e. will not need transportation again, rendering vehicle withholding irrelevant. Outside Economics, Nansubuga and Kowalkowski (2021) review literature analysing the decision to adopt carsharing, usually based on consumer surveys rather than theory.

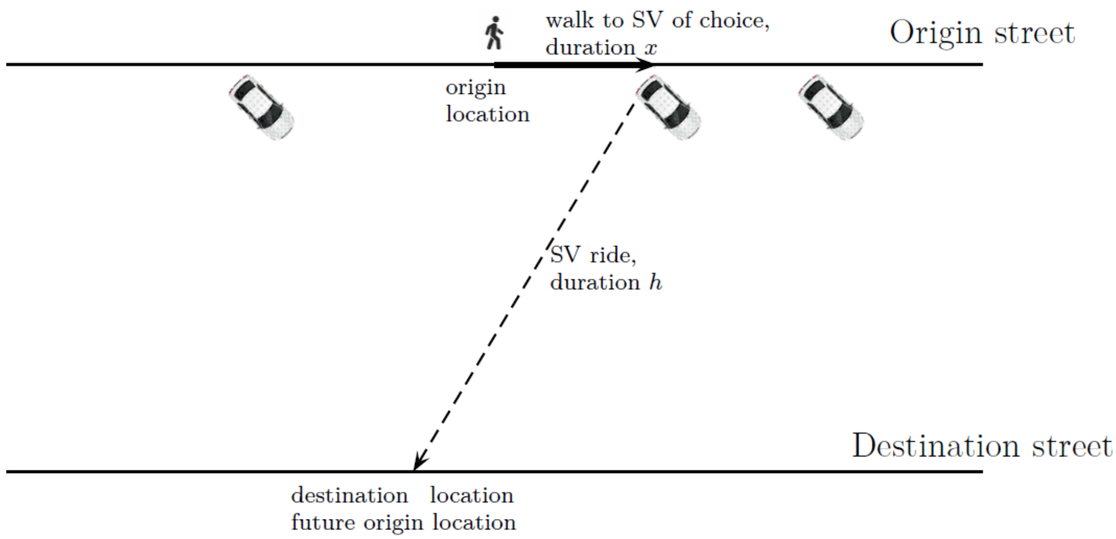

Figure 1: Illustration of a typical trip. Reproduced from Zakharenko (2023b). Vehicle image courtesy of Macrovector/Freepik.

2. The model

2.1. Definitions

The model is based on Zakharenko (2023b) with some modifications. There is a continuum of individuals who live in infinite continuous time and travel between two infinite linear streets repeatedly, using a vehicle. The precise location of the next trip origin is the same as the location of the previous destination. All vehicles are (potentially) shared and are free-floating, i.e. can be released for the use by others at any time and location. A spatial visualisation of the model is illustrated on Figure 1. At the end of each trip, travellers decide whether to release the vehicle or withhold it for future use.

Traveller departure for the next trip is a Poisson process that cannot be planned in advance, such that the expected duration of stay between trips is τ. The mass of travelers who depart, per hour per km of linear street space, is exogenously given by L. The exact location at the destination street is also random and uniformly distributed across space, which means that the distribution of vehicles along each street is also uniform. Given these assumptions, the total amount of vehicle-kilometers travelled, as well as the cost of such

Notation	Description	Units
λ	fraction of travellers sharing vehicles	
μ	density of vacant vehicles	veh*/ kmS**
τ	mean duration of stay at destination	h
ϕ	vehicle standing cost	\$/h/veh
C	Social cost of transportation	\$/h/kmS
I	Individual probability of vehicle withholding	
L	travel demand	per***/ h/ kmS
q	Rate of SV reservation by walkers	$1/h$
w	cost of walking	\$/h

Table 1: Notational glossary. *per vehicle or number of vehicles; **km of street space; *** number of persons.

travel, are an exogenous part of social transportation cost and are not modelled explicitly.

The social cost of parked vehicles is ϕ per hour; this includes the vehicle standing costs and the social cost of parking.[2] Parked vehicles can be in one of the three states: withheld, vacant, and reserved, explained below.

When a traveller chooses to withhold a vehicle at the end of the previous trip, the vehicle remains in this state until the next trip, that is, for τ units of time in expectation. When the vehicle is released, it becomes vacant and available for anyone. The endogenous density of vacant vehicles is denoted by μ.

Immediately before the next trip, travellers who previously released their vehicles search for a vehicle again. With positive probability, previously used vehicle is still available at the same location, so the traveller can use it with zero walking cost involved. With the remaining probability, the previous vehicle is no longer available, and another one has to be found. Almost surely an alternative vehicle is some walking distance away; then such travellers are denoted as *walkers*. In social optimum, walkers will always use the nearest available vehicle, which is $\frac{1}{2\mu}$ units of time (and also units of space, assuming unitary walking speed) away in

[2]In previous variants of this model, Zakharenko (2023a) and Zakharenko (2023b), these two elements were modelled separately.

expectation.[3] The vehicle has to be reserved while the traveller is walking towards it. The incurred walking cost $w \geq 0$ per hour is heterogenous with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$.

2.2. Individual release decision

To keep the model tractable, I assume that the individual release decision can depend only on the time-invariant individual walking cost w. It cannot depend on transitory parameters such as availability of other vehicles at the time of release. It cannot be revised before the departure for the next trip. Travellers without a withheld vehicle, when searching for vacant vehicles, know only vehicle locations and cannot base their decisions on past history of these vehicles. These assumptions about information (un)available to travellers are consistent with the existing practice of SV operations.

Given above assumptions, denote by $I(w)$ the endogenous probability that a traveller with walking cost w chooses to withhold their vehicle at the end of their trip. Denote by q the endogenous Poisson rate of vacant vehicle booking by travellers who had to walk, i.e. who were unable to use their previous vehicle.

Unused (parked) vehicles incur a social cost ϕ per hour and belong to one of three categories:

- Withheld. With arrival rate L and expected parking duration τ, the density of withheld vehicles per km of street space is $L\tau \int_w I(w)f(w)\mathrm{d}w$.
- Vacant. The density is denoted by μ. The flow of travellers that arrive and release their vehicles is $L \int_w (1 - I(w))f(w)\mathrm{d}w$. For every released vehicle, the rate of their booking is $\frac{1}{\tau}$ by the same traveller and q by others, hence the expected duration of

[3] The coefficient 2 here is because vehicles can be searched in both directions along the street.

vacancy is $\frac{1}{\frac{1}{\tau}+q} = \frac{\tau}{1+q\tau}$. The density of vacant vehicles then satisfies

$$\mu = \frac{L\tau}{1+q\tau}\int_w (1-I(w))f(w)\mathrm{d}w. \tag{1}$$

- Reserved by walkers. The flow of travellers who originate their trip without a previously withheld vehicle is $L\int_w(1-I(w))f(w)\mathrm{d}w$. For every such traveller who stayed t hours since the previous trip, the probability that the previous vehicle is no longer available is $1-\exp(-qt)$. Since t is random and exponentially distributed with p.d.f. $\frac{1}{\tau}\exp\left(-\frac{t}{\tau}\right)$, the ex-ante probability of having to search for another vehicle is

  $$\frac{1}{\tau}\int_{t=0}^{\infty}(1-\exp(-qt))\exp\left(-\frac{t}{\tau}\right)\mathrm{d}t = \frac{q\tau}{1+q\tau}.$$

  As outlined in Section 2.1, the expected walking time is $\frac{1}{2\mu}$. Given all of the above, the density of reserved vehicles is $\frac{L}{2\mu}\frac{q\tau}{1+q\tau}\int_w(1-I(w))f(w)\mathrm{d}w$, and the walking cost incurred by those who reserved them is $\frac{L}{2\mu}\frac{q\tau}{1+q\tau}\int_w(1-I(w))wf(w)\mathrm{d}w$ per hour.

Given the above, we can formulate the social cost of all unused (parked) vehicles (including the associated walking cost) as follows:

$$C = \phi\left(L\tau\int_w I(w)f(w)\mathrm{d}w+\mu\right)+\frac{L}{2\mu}\frac{q\tau}{1+q\tau}\int_w(1-I(w))(w+\phi)f(w)\mathrm{d}w. \tag{2}$$

The social cost of transportation also includes the cost of vehicles in transit. However, given the assumptions of the model, this part of social cost does not depend on any endogenous parameters in the model and is therefore omitted from further analysis.

Denote by *quasi-shared* state the environment saturated with vehicles, so that a released vehicle cannot be picked up by anyone other than last customer: $q=0$. Then, from (1), $\mu = L\tau\int_w(1-I(w))f(w)\mathrm{d}w$ and, from (2), the social cost is a constant at $C=\phi\tau L$.

Intuitively, with one vehicle per traveller, it does not matter whether a vehicle is withheld or released, as it will always be used by the same traveller.

The *truly shared* state, in contrast, implies fewer vehicles than travellers and therefore $q > 0$. Then, from (1), we can show that the term involving q in (2) is equal to

$$\frac{q\tau}{1+q\tau} = 1 - \frac{\mu}{\tau L \int_w (1 - I(w)) f(w) \mathrm{d}w} > 0, \tag{3}$$

and (2) can be solved for q as

$$C = \phi \left(L\tau \int_w I(w) f(w) \mathrm{d}w + \mu \right) + \frac{L}{2\mu} \int_w (1 - I(w))(w + \phi) f(w) \mathrm{d}w - \frac{\phi + \bar{w}(I(\cdot))}{2\tau}, \tag{4}$$

where

$$\bar{w}(I(\cdot)) = \frac{\int_w (1 - I(w)) w f(w) \mathrm{d}w}{\int_w (1 - I(w)) f(w) \mathrm{d}w} \tag{5}$$

is the mean walking cost among travellers releasing their vehicles.

The socially optimal withholding decision $I(w)$ is found by comparing to zero the following:

$$\frac{\mathrm{d}C}{\mathrm{d}I(w)} = L f(w) \mathrm{d}w \left[-\frac{w + \phi}{2\mu} + \frac{w - \bar{w}(I(\cdot))}{2L\tau \int_{w'} (1 - I(w')) f(w') \mathrm{d}w'} + \tau\phi \right]. \tag{6}$$

It is optimal to withhold ($I(w) = 1$) if (6) is negative, and to release ($I(w) = 0$) if it is positive. How does this decision depend on the individual walking cost w? Differentiation of the term in square brackets in (6) w.r.t. w yields

$$-\frac{1}{2\mu} + \frac{1}{2\tau L \int_{w'} (1 - I(w')) f(w') \mathrm{d}w'},$$

which is negative by (3). This proves the intuitive result that a higher walking cost w decreases $\frac{\mathrm{d}C}{\mathrm{d}I(w)}$, making it negative for sufficiently large w.

2.3. Social optimum

Given the latter conclusion, there exists $\hat{w} \in [0, \infty)$ such that all travellers with $w < \hat{w}$ release their vehicle and those with $w > \hat{w}$ withhold it.[4] The rest of the paper operates with this threshold $\hat{w}$ rather than individual decisions $I(w)$ to save on notation. Denote by $\lambda = F(\hat{w})$ the endogenous fraction of traveller population who release their vehicle after each trip. The mean walking cost (5) is redefined as:

$$\bar{w}(\lambda) = \frac{1}{\lambda} \int_0^{F^{-1}(\lambda)} w f(w) \mathrm{d}w. \tag{7}$$

The social transportation cost function (4) is redefined as

$$C(\lambda, \mu) = \max \left[\frac{\lambda L}{2\mu} - \frac{1}{2\tau}, 0 \right] (\phi + \bar{w}(\lambda)) + \mu\phi + (1 - \lambda)\tau\phi L \to \min_{\lambda, \mu} \tag{8}$$

The condition for a truly shared state (3) is redefined as

$$\mu < \lambda \tau L. \tag{9}$$

The first-order conditions of local optimum are:

$$\frac{\partial C}{\partial \lambda} = \frac{L}{2\mu}(\phi + \hat{w}) - \frac{1}{2\tau} \frac{\hat{w} - \bar{w}}{\lambda} - \tau\phi L \begin{cases} = 0, \lambda < 1, \\ \leq 0, \lambda = 1, \end{cases} \tag{10}$$

$$\frac{\partial C}{\partial \mu} = -\frac{\lambda L}{2\mu^2}(\phi + \bar{w}) + \phi = 0. \tag{11}$$

[4] Those with $w = \hat{w}$ are indifferent and may play a mixed strategy, which will affect aggregate outcomes only if $\hat{w}$ is a mass point of the walking cost distribution.

The elements of the Hessian H are

$$\frac{\partial^2 C}{\partial \lambda^2} = \frac{1}{2}\left[\frac{L}{\mu} - \frac{1}{\lambda\tau}\right]\frac{1}{f(\hat{w})} + \frac{\hat{w}-\bar{w}}{\lambda^2\tau} > 0, \tag{12}$$

$$\frac{\partial^2 C}{\partial \lambda \mathrm{d}\mu} = -\frac{L}{2\mu^2}(\phi + \hat{w}) < 0, \tag{13}$$

$$\frac{\partial^2 C}{\partial \mu^2} = \frac{\lambda L}{\mu^3}(\phi + \bar{w}) > 0. \tag{14}$$

Because the diagonal elements of the Hessian are positive, the curve in the $\{\mu, \lambda\}$ space defined by (10) held with equality is optimal (cost-minimising) λ for given μ and is labeled *the demand curve*, $\lambda_D(\mu)$. Likewise, the curve defined by (11) is optimal μ for a given λ and is labeled *the supply curve*, $\lambda_S(\mu)$. The intersection of demand and supply is an interior local optimum if the Hessian is positive-definite, and is a saddle point otherwise.

Lemma 1. *Both demand and supply are non-negatively sloped. Their intersection is a local minimum if the demand is flatter (with respect to μ) than supply, and is a saddle point otherwise.*

Proof. From the implicit function theorem, $\frac{\mathrm{d}\lambda_D}{\mathrm{d}\mu} = -\frac{\partial^2 C \backslash \partial\lambda\partial\mu}{\partial^2 C \backslash \partial\lambda^2}$. Likewise, the slope of the supply curve is $\frac{\mathrm{d}\lambda_S}{\mathrm{d}\mu} = -\frac{\partial^2 C \backslash \partial\mu^2}{\partial^2 C \backslash \partial\lambda\partial\mu}$. Both are positive due properties of the Hessian. Multiplying both ratios by positive quantity $-\frac{\partial^2 C}{\partial\lambda \mathrm{d}\mu}\frac{\partial^2 C}{\partial\lambda^2}$, we conclude that $\frac{\mathrm{d}\lambda_D}{\mathrm{d}\mu} < \frac{\mathrm{d}\lambda_S}{\mathrm{d}\mu}$ iff $\left(\frac{\partial^2 C}{\partial\lambda \mathrm{d}\mu}\right)^2 < \frac{\partial^2 C}{\partial\lambda^2}\frac{\partial^2 C}{\partial\mu^2}$, i.e. the Hessian is positive-definite and the intersection point is a local minimum. The reverse inequality $\frac{\mathrm{d}\lambda_D}{\mathrm{d}\mu} > \frac{\mathrm{d}\lambda_S}{\mathrm{d}\mu}$ implies the Hessian is negative-definite. ■

Zakharenko (2023b) studies a special case of the above model with homogenous waking cost w. In the notation of the current paper, homogenous w means $\hat{w} = \bar{w} = w$, meaning that the demand curve (10) becomes $\frac{L}{2\mu}(\phi + w) - \tau\phi L \begin{cases} = 0, \lambda < 1, \\ \leq 0, \lambda = 1. \end{cases}$ In other words, the demand curve is vertical (λ_D takes any value) at $\mu = \frac{\phi+w}{2\tau\phi}$, while $\lambda_D(\mu) = 1, \forall\mu > \frac{\phi+w}{2\tau\phi}$.[5]

The fully shared local optimum is where $\lambda = 1$, while μ is defined by the supply curve.

[5]This result is repeated in formula (7) in Zakharenko (2023b), after setting to zero the parameters not introduced in the current paper.

2.4. Quasi-shared critical points

What is the relationship between the quasi-shared curve $\mu = \lambda\tau L$, demand, and supply curves? By inserting the latter equation for μ into (10,11), we find that the demand and supply curves intersect the quasi-shared line at the same point(s) given by

$$\phi + \bar{w}(\lambda) = 2\lambda\tau^2\phi L. \tag{15}$$

In other words, the equations $\lambda_D(\lambda\tau L) = \lambda$ and $\lambda_S(\lambda\tau L) = \lambda$ have the same solution(s), if any. Because the left-hand side of (15) is strictly positive, such point(s), if they exist, correspond to strictly positive values of λ and μ.

Furthermore, the determinant of the Hessian at such points is given by

$$\frac{1}{\lambda^4\tau^4L^2}\left[(\hat{w} - \bar{w})(\phi + \bar{w}) - \frac{1}{4}(\phi + \hat{w})^2\right].$$

This a non-positive quantity, and is strictly negative as long as $\bar{w} \neq \frac{1}{2}(\hat{w} - \phi)$. Thus, any quasi-shared critical point is a saddle point of $C(\lambda, \mu)$.

2.5. Truly shared critical points

This section studies interior ($\lambda < 1$) Truly Shared (satisfying (9)) Critical Points, or TSCP. First, solve (11) for $\bar{w}$ as follows: $\bar{w} = \frac{2\mu^2\phi}{\lambda L} - \phi$. Next, insert this solution into (10) (for interior λ), to obtain

$$\frac{L}{2\mu}(\phi + \hat{w})\left(1 - \frac{\mu}{\lambda\tau L}\right) - \phi\tau L\left(1 - \frac{\mu^2}{\lambda^2\tau^2L^2}\right) = 0.$$

Next, divide both sides by term $1 - \frac{\mu}{\lambda\tau L}$ (which is positive due to (9)):

$$\frac{L}{2\mu}(\phi + \hat{w}) - \tau\phi L\left(1 + \frac{\mu}{\lambda\tau L}\right) = 0.$$

Given the latter expression, we can replace the term $\frac{L}{2\mu}(\phi + \hat{w})$ in (10) (for interior λ) by $\tau\phi L \left(1 + \frac{\mu}{\lambda\tau L}\right) = \tau\phi L + \frac{\mu\phi}{\lambda}$, to arrive at the following property of a TSCP:

$$2\mu\tau\phi = \hat{w} - \bar{w}. \tag{16}$$

By solving (16) for μ and inserting into (11), we arrive at a univariate equation as necessary condition of a TSCP:

$$2\lambda\tau^2\phi(\phi + \bar{w}(\lambda))L = (\hat{w}(\lambda) - \bar{w}(\lambda))^2. \tag{17}$$

Furthermore, (9) and (16) jointly imply $\hat{w} - \bar{w} < 2\lambda\tau^2\phi L$, which together with (17) implies $\hat{w} - \bar{w} > \bar{w} + \phi$, or

$$\hat{w} - 2\bar{w} > \phi, \tag{18}$$

as a second necessary condition of a TSCP. (17) and (18) are jointly sufficient for a TSCP.

Note that (18) is quite restrictive and requires a sufficiently large variation in walking costs w. For example, any uniform distribution with a positive support does not satisfy this property, as $\hat{w} - 2\bar{w} \leq 0$ (with equality if the lower bound is zero, and inequality if it is positive). This means that a uniform distribution of walking costs is not variable enough to justify withholding a vehicle by anyone.

Figure 2 visualises the social cost of transportation and optimal λ and μ for the cases where everyone has the same walking cost (left panel) and for a triangular distribution of walking cost with linearly decreasing density $f(\cdot)$ (right panel).

3. Maximum withholding

This section plays devil's advocate to construct a social optimum that maximises the share of travelers who should withhold their vehicle. Note that in a quasi-shared state, the number of vehicles matches the number of travellers, meaning all of them are indifferent

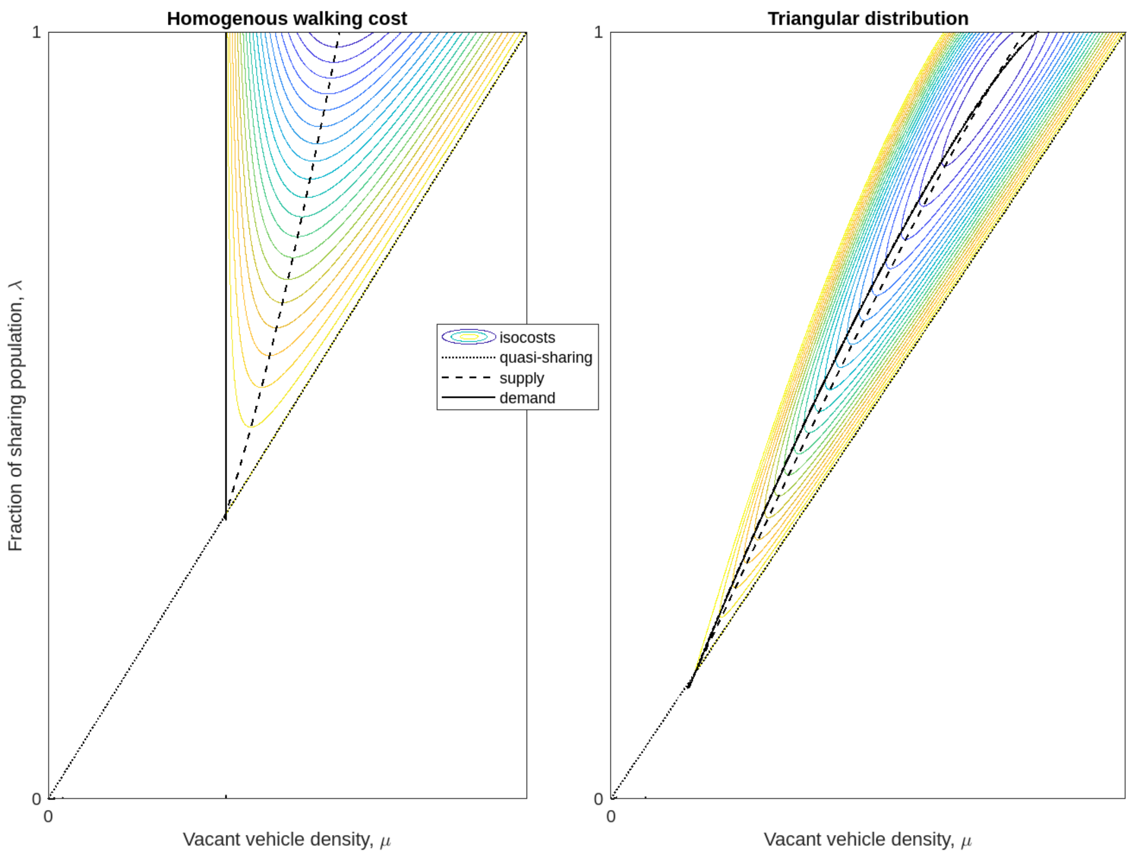

Figure 2: Examples of social cost of transportation as function of μ and λ.

between withholding and not (because the last used vehicle will remain available for future use anyway). This section focuses on cases where withholding is strictly preferred for some travelers, which is only possible when some vehicles are truly shared. The objective of this section is therefore to characterise the lowest possible λ in a truly shared global minimum of the social cost function (8).

With unbounded support of walking cost w, one can construct an example with vanishingly small fraction λ of travelers sharing in the social optimum. It is quite plausible to assume however that there exists an upper bound on the walking cost, because travelers can use taxi or other means of transportation to reach the shared vehicle. Some SV operators in Moscow, for example, connect their customers to a taxi service to do so, helping to reduce the transaction costs of finding transportation to their SV. We will therefore assume an upper bound of the walking cost, denoted w_H. Playing devil's advocate again, we assume the lower bound of the distribution is zero, as large variance in w is essential for socially optimal vehicle withholding. Intuitively, some people may walk certain distance per day anyway, so for them there is no opportunity cost of walking towards an SV.

3.1. Binomial distribution

To maximise the chance that some pair $\{\lambda_L, \mu\}$ satisfying (9) and $\lambda_L < 1$ is a global minimum of social cost (8), one wants to minimise $\bar{w}(\lambda_L)$ in (8). That is equivalent to assuming that fraction λ_L of travelers have the lowest possible walking cost of zero. Furthermore, we also want the social cost to be as *high* as possible at all alternative local minima with $\lambda > \lambda_L$. For that, we need to maximise $\bar{w}(\lambda), \lambda > \lambda_L$, by assuming that the remaining fraction $1-\lambda_L$ of travelers have the highest possible walking cost w_H. Such binary distribution maximises the fraction of travelers who can withhold their vehicle in the social optimum.

The demand curve for such binary distribution is as follows.

- For $\lambda < \lambda_L$, we have that $\hat{w} = \bar{w} = 0$; the demand curve (10) is vertical at $\mu = \frac{1}{2\tau}$.

For candidate social optimum at λ_L to be truly shared (i.e. for (9) to hold at λ_L), the following must be true:

$$\lambda_L > \frac{1}{2\tau^2 L}. \tag{19}$$

- At $\lambda = \lambda_L$, $\hat{w}$ increases from zero to w_H, while $\bar{w}$ remains at zero. The demand curve is horizontal, with μ ranging from $\frac{1}{2\tau}$ to $\mu_1^D \equiv \frac{\lambda_L \tau L(\phi+w_H)}{w_H+2\lambda_L\tau^2\phi L}$. This whole segment is truly shared, as long as (19) is true.

- For $\lambda > \lambda_L$, $\hat{w} = w_H$ and $\bar{w} = \left(1 - \frac{\lambda_L}{\lambda}\right) w_H$; the demand curve takes form

$$\lambda_D(\mu)^2 = \begin{cases} \frac{\lambda_L \mu w_H}{\tau L(\phi+w_H-2\mu\tau\phi)}, & \mu \in \left[\mu_1^D, \mu_2^D\right); \\ 1, & \mu > \mu_2^D, \end{cases} \tag{20}$$

  where $\mu_2^D \equiv \frac{\tau L(\phi+w_H)}{\lambda_L w_H + 2\tau^2\phi L}$.

Using the above expressions for $\bar{w}(\lambda)$, we can also find the supply curve (11):

$$\lambda_S(\mu) = \begin{cases} \frac{2\mu^2}{L}, & \mu \leq \mu_1^S \equiv \sqrt{\frac{1}{2}\lambda_L L}; \\ \frac{2\mu^2\phi}{L(\phi+w_H)} + \frac{\lambda_L w_H}{\phi+w_H}, & \mu > \mu_1^S. \end{cases} \tag{21}$$

Figure 3 illustrates these findings.

3.2. Lower optimum

This section investigates properties of a local optimum where part of travellers withhold their vehicles, which we denote as a *lower optimum*.

For λ_L to be indeed a locally optimal λ, it is necessary that the demand and supply curves intersect at λ_L; from (21), such intersection must occur at μ equal to μ_1^S. Since the demand curve has a horizontal segment at λ_L, the supply curve should cross through that segment: $\frac{1}{2\tau} < \mu_1^S < \mu_1^D$. The first inequality $\frac{1}{2\tau} < \mu_1^S$ follows from (19), while the second

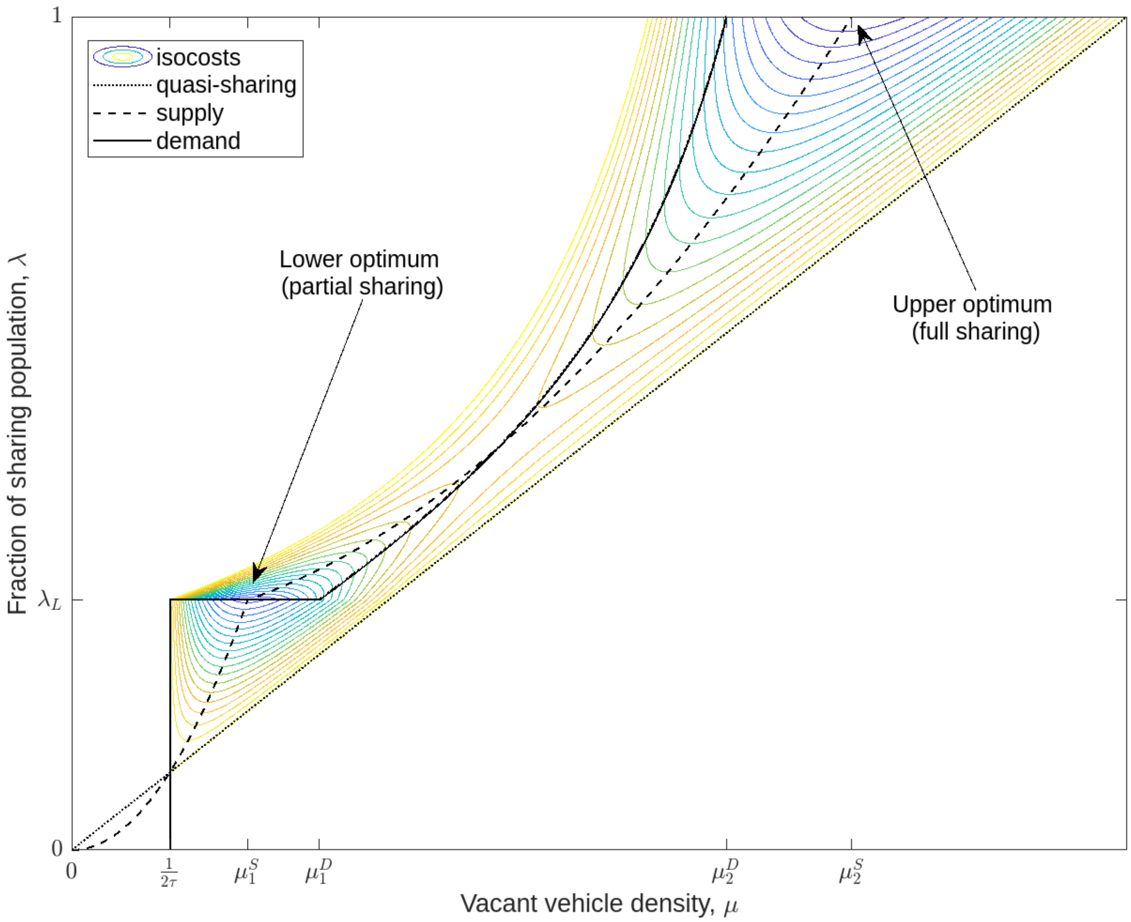

Figure 3: Typical local optima of social welfare under a binomial distribution of walking costs

inequality $\mu_1^S < \mu_1^D$ requires that

$$w_H > \sqrt{2\lambda_L L}\phi\tau \tag{22}$$

as a necessary condition of local optimum existence. Note that the right-hand side of (22) is greater than ϕ by (19): vehicle withholding is not optimal for anyone when the maximum walking cost w_H is lower or equal than the vehicle cost ϕ.

If the above conditions are met, the point $\{\lambda_L, \mu_1^S\}$ is indeed a local optimum, by Lemma 1 and by the fact that demand is horizontal while supply is upward-sloping with respect to μ.

Next, we investigate other local optima at higher values of λ, and specify conditions under which the point $\{\lambda_L, \mu_1^S\}$ is the global optimum.

Proposition 1. *Any combination $\{\lambda, \mu\}$ for $\lambda \in (\lambda_L, 1)$ cannot be a local minimum of (8).*

Proof. Since a local interior minimum is a TSCP, it must satisfy conditions (16) and (18). For the bilateral distribution of walking costs studied in the current section, for $\lambda > \lambda_L$, these two conditions are

$$2\mu\tau\phi = \frac{\lambda_L}{\lambda} w_H, \tag{23}$$

$$\left(2\frac{\lambda_L}{\lambda} - 1\right) w_H > \phi. \tag{24}$$

Because the relationship between λ and μ in (23) is negative, while in both demand (20) and supply (21) such relationship is positive, all these conditions can be simultaneously met in at most one point satisfying $\lambda \in (\lambda_L, 1)$. We will refer to this as the "candidate point".

Next, we calculate the Hessian and its determinant at the candidate point. Due to atomic distribution of the walking cost, $f(\hat{w}) = f(w_H) = \infty$ in (12) for $\lambda \in (\lambda_L, 1)$, hence (12,13,14) can be rewritten as

$$\frac{\partial^2 C}{\partial \lambda^2} = \frac{\lambda_L w_H}{\lambda^3 \tau},$$
$$\frac{\partial^2 C}{\partial \lambda \mathrm{d}\mu} = -\frac{L}{2\mu^2}(\phi + w_H),$$
$$\frac{\partial^2 C}{\partial \mu^2} = \frac{\lambda L}{\mu^3}\left(\phi + w_H - \frac{\lambda_L}{\lambda} w_H\right).$$

The determinant of the Hessian at the candidate point is then equal to

$$\begin{aligned}
\det H &= \frac{\lambda_L w_H}{\lambda^3 \mu^3 \tau}\left(\lambda L(\phi + w_H) - \lambda_L L w_H\right) - \left(\frac{L}{2\mu^2}(\phi + w_H)\right)^2 \\
&\underbrace{=}_{(21)\text{ for }\mu > \mu_1^S} \frac{2\lambda_L w_H \phi}{\lambda^3 \mu \tau} - \left(\frac{L}{2\mu^2}(\phi + w_H)\right)^2 \\
&\underbrace{=}_{(23)} \left(\frac{2\phi}{\lambda}\right)^2 - \left(\frac{L}{2\mu^2}(\phi + w_H)\right)^2.
\end{aligned}$$

The sign of $\det H$ is therefore the same as the sign of

$$\frac{2\phi}{\lambda} - \frac{L}{2\mu^2}(\phi + w_H) \underbrace{=}_{(21)\text{ for }\mu > \mu_1^S} \frac{2\phi}{\lambda} - \frac{\phi}{\lambda - \frac{\lambda_L w_H}{\phi + w_H}} \underbrace{<}_{(24)} \frac{2\phi}{\lambda} - \frac{\phi}{\lambda - \frac{1}{2}\lambda} = 0.$$

Thus, the determinant of the Hessian at the candidate point is negative, meaning that such point, if it exists, is a saddle point rather than a local minimum of social cost. ■

3.3. The global optimum and maximum withholding

The only alternative local optimum is therefore a fully shared one (with no one withholding vehicles), where $\lambda = 1$ and $\mu = \mu_2^S$ that solves $\lambda_S(\mu_2^S) = 1$, which we refer to as the *upper optimum.* Then, any withholding can exist in global social optimum if and only if the local optimum defined in section 3.2 delivers a lower social transportation cost than that of a fully shared optimum:

$$C(\lambda_L, \mu_1^S) \leq C(1, \mu_2^S). \tag{25}$$

The aim of this section is to find the lowest λ_L that satisfies (25), which in turn ensures that the maximal fraction $1 - \lambda_L$ of travellers are withholding their vehicles.

The value of minimal λ_L depends on exogenous model parameters L and τ, among others. Both of these positively affect the demand for shared mobility: a higher demand density L means there are more people who can share, while a higher duration of stay between trips τ increases the cost of withholding. Exposition of results that follow can be simplified and made more intuitive by replacing these two parameters by new notation. Specifically, denote by *sharers* the fraction λ_L of travellers actually sharing vehicles in the lower optimum. Denote by n the socially optimal customer-to-vehicle ratio among the sharers; it is defined as ratio of $\lambda_L L\tau$ (the density of vehicles withheld by the sharers, if they chose to do so) to μ_1^S (the actual density of vehicles released by sharers), $n\mu_1^S \equiv \lambda_L L\tau$. By comparing the latter against the definition of μ_1^S in (21), we can backtrack the demand density L that results in that specific value of n, and also find the socially optimal μ, as follows: $\mu_1^S = \frac{n}{2\tau}$ and $L = \frac{n^2}{2\lambda_L \tau^2}$. By substituting the latter into (19) and (22), these constraints can be rewritten as $n > 1$ and $w_H > n\phi$, respectively, as necessary conditions of existence of a local optimum with vehicle sharing by some travellers and withholding by others. The two inequalities impose bounds on the traveller-to-vehicle ratio: $n \in \left(1, \frac{w_H}{\phi}\right)$. Outside of these bounds, $n = 1$ implies quasi- rather than true sharing, while $n \geq \frac{w_H}{\phi}$ means high enough demand density so that no vehicle withholding can be optimal.

Next, we can calculate the social transportation cost at the designated global optimum, i.e. at the lower optimum, as follows (cf.(8), recalling that $\bar{w}(\lambda_L) = 0$):

$$C_0(\lambda_L, n) = C(\lambda_L, \mu_1^S) = \frac{1}{2}\left[\frac{\lambda_L L}{\mu_1^S} - \frac{1}{\tau}\right]\phi + \mu_1^S\phi + (1 - \lambda_L)\tau\phi L = \frac{\phi}{\tau}\left(n - \frac{1}{2} + \frac{1 - \lambda_L}{\lambda_L}\frac{n^2}{2}\right). \tag{26}$$

The social transportation cost at the competing upper optimum is (noting that $\mu_2^S = \frac{n}{2\tau}\sqrt{\frac{\phi+w_H-\lambda_L w_H}{\lambda_L\phi}}$ and recalling that $\bar{w}(1) = (1-\lambda_L)w_H$)

$$C_1(\lambda_L, n) = C(1, \mu_2^S) = \frac{1}{2}\left[\frac{L}{\mu_2^S} - \frac{1}{\tau}\right](\phi + (1-\lambda_L)w_H) + \mu_2^S\phi \tag{27}$$

$$= \frac{n}{\tau}\sqrt{\frac{\phi}{\lambda_L}(\phi + (1-\lambda_L)w_H)} - \frac{\phi + (1-\lambda_L)w_H}{2\tau}. \tag{28}$$

Finally, we need to find the minimal λ_L that satisfies (cf.(25))

$$C_0(\lambda_L, n) \leq C_1(\lambda_L, n). \tag{29}$$

Denote such minimal λ_L by λ_M. Note that $C_0(1, n) = C_1(1, n)$, by construction. Also note that, as $\lambda_L \to 0$, $\lambda_L C_0(\lambda_L, n) \to \frac{\phi n^2}{2\tau}$ while $\lambda_L C_1(\lambda_L, n) \to 0$, meaning that (29) is violated for sufficiently small λ_L, and λ_M is strictly positive. We also have that $C_0\left(\frac{\phi}{w_H}, 1\right) = C_1\left(\frac{\phi}{w_H}, 1\right)$.

Proposition 2. *The function $\lambda_M(n)$ is implicitly defined by*

$$n = \frac{\sqrt{\lambda}\left(1+\sqrt{\lambda}\right)w_H/\phi}{\sqrt{1+(1-\lambda)w_H/\phi}+1}, \tag{30}$$

and is monotonically increasing from $\lambda_M(1) = \frac{\phi}{w_H}$ to $\lambda_M(w_H/\phi) = 1$.

Monotonicity and boundary conditions are straightforward to verify from (30). Appendix Appendix A proves that this function is indeed $\lambda_M(\cdot)$. Figure 4 compares $C_0(\lambda_L, n)$ and $C_1(\lambda_L, n)$ and visualises the function $\lambda_M(n)$.

3.4. Calibration

To get some sense of the magnitude of λ_M, this section conducts its crude calibration for the case of London.

First, I calibrate the maximum "walking cost" w_H, on the assumption that travellers with any higher walking cost travel to an SV of their choice by taxi. Calibration is made

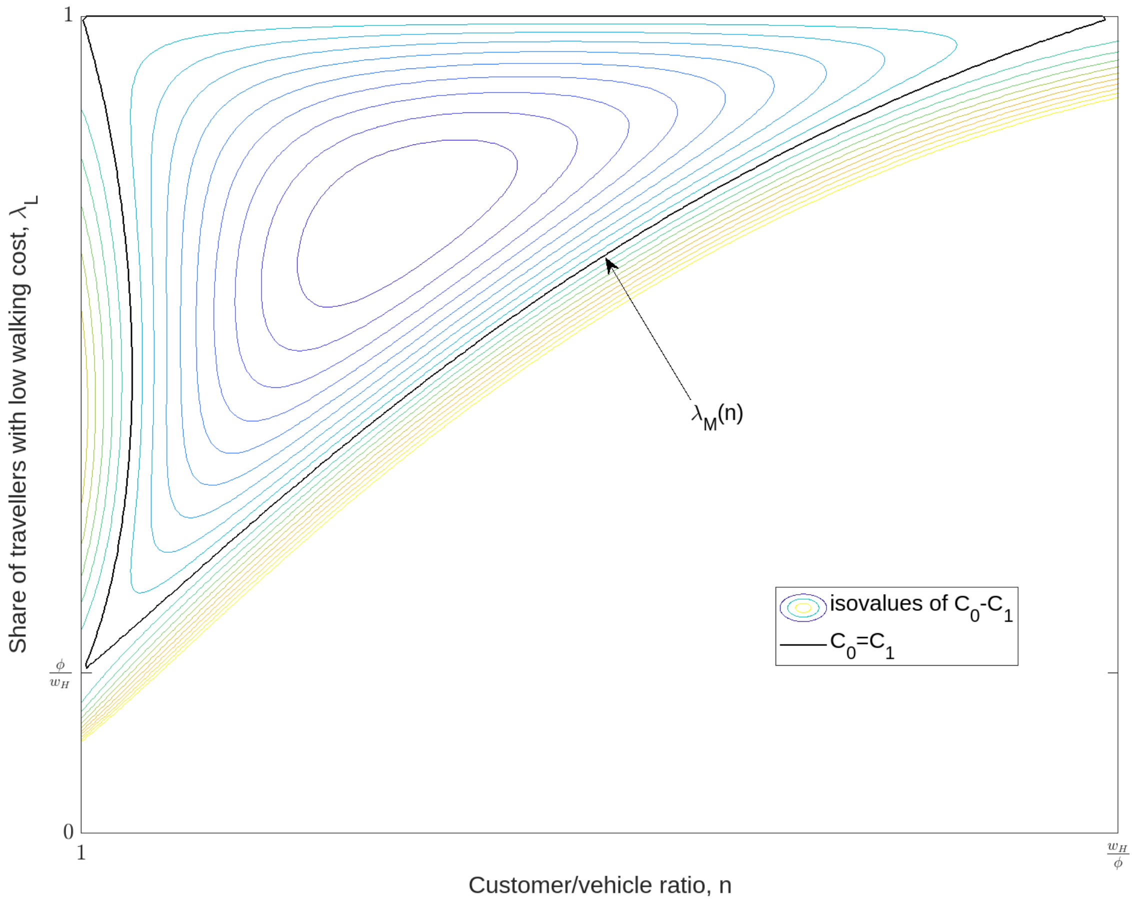

Figure 4: Comparison of $C_0(\lambda_L, n)$ and $C_1(\lambda_L, n)$

on a further assumption that the nearest SV is 2km away, which costs £7.6 to travel by taxi according to London "taxi tariff 1"[6] Assuming further that the trip takes 12min (5min wait for taxi and 7min ride), with traveller's maximal value of time of £1/min, the total cost of a search session is 12+7.6+12ϕ=£19.6+12ϕ. The last element here is the cost of SV reservation during the 12min during the travel time towards it.

Because w_H is defined as the maximal value of *walking* time, we calculate it for a traveller who *walks* 2km to an SV and incurs a total cost of £19.6+12ϕ in the process. Such cost is $w_H + \phi$ per walking minute; assuming it takes 24min to walk 2km, we have $w_H + \phi = (19.6 + 12\phi)/24 = 0.8167 + 0.5\phi$ per min.

The vehicle standing cost ϕ is calibrated at £540/month (based on a typical cost of compact-size "vehicle subscription" cost plus insurance). Assuming non-trivial transportation demand exists for 10 hours per day, this monthly standing cost corresponds to £1.8/h or £0.03/min. In addition to the capital cost of the vehicle *per se*, we also add the social cost of parking, as unused vehicles create negative congestion externalities.[7] Zakharenko (2016) develops a method to calculate the social cost of parking; this method has been applied to Melbourne (van Ommeren et al., 2021) and Stockholm (Eliasson and Börjesson, 2022). Because no similar estimate was done for London, I proxy it (on the conservative side) by the minimal fee of £0.02/min.[8] Thus, $\phi =$£0.05/min.

Based on above calibration, the value of maximal "walking cost" is calibrated at $w_H = (19.6 + .05 \times 12)/24 - .05 =$£0.7917/min. The ratio $\frac{w_H}{\phi}$ is then $0.7917/0.05 = 15.83$.

The minimal λ_L also depends on equilibrium n. Jochem et al. (2020), based on stated consumer preferences, estimate that a shared car replaces $n = 13.3$ private cars in London.

[6] £3.8 for initial 190.8m, and £0.2 for every additional 95.4m. Reference: https://tfl.gov.uk/modes/taxis-and-minicabs/taxi-fares/tariffs, accessed on 29.01.2024.

[7] Zakharenko (2023a) and Zakharenko (2023b) use a separate notation for parking social cost; in the current paper, it is integrated with ϕ for notational brevity.

[8] This is based on the £1.1/h minimal fee, as published at https://www.justpark.com/uk/parking/london/, accessed on 30.01.2024.

Then, $\lambda_M(13.3)$ is given by (30) and equal to 0.9561. Zakharenko (2023a), by comparing observed usage intensity of shared vs. private vehicles, makes a more conservative estimate of $n = 6$, which corresponds to a minimal λ_L of 0.6111. The latter estimate implies that at most 38.89% of London automobile users, those with highest walking costs, should use their vehicles exclusively, while the remaining 61.11% should share. This estimate is based on the assumption of extreme walking cost inequality, so that the 38.89% have such a high walking cost that they would need to use taxi to reach a vacant shared vehicle, while the other 61.11% have zero walking cost. Under any other distribution of walking cost, the share of those withholding a vehicle would be lower than 38.89%, given the above calibration. In particular, increasing the share of high-walking-cost population beyond 38.89% would mean that they should optimally stop withholding their vehicles, and share them alongside others.

4. Conclusion

This paper investigates the theoretical justifications for vehicle withholding, i.e. preventing a vacant vehicle from being used by others, for the purpose of own future use. It is shown that inequality in the cost of search for alternative vehicles can justify withholding by a fraction of individuals with the highest search cost; a theoretical upper bound on such fraction is established, and calibrated for the city of London.

Future research may explore other potential justifications for vehicle withholding. In particular, other dimensions of traveller heterogeneity can be analysed. For example people may differ in how frequently they demand travel, so there is a question of whether vehicle withholding is more justified for those travelling more frequently. Another possible dimension is spatial inequality: some locations are more dense than others, so one may wonder whether withholding is more justified in low-density areas with fewer opportunities for sharing. There is also a question of open withholding strategy in the presence of multiple types of vehicles (e.g. luxury/economy, large/small, etc.), and with travellers having heterogenous preferences over these types. Can it be optimal to withhold a vehicle of a certain type by a certain traveller, for example when an alternative vehicle of the same type is difficult to find?

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Appendix A. Proof of Proposition 2

Denote $w_p \equiv \frac{w_H}{\phi} > 1$[9] and define by H the difference of social transportation cost in the lower and the upper optima, multiplied by a positive coefficient:

$$\begin{aligned} H(\lambda, n) &\equiv \frac{\lambda\tau}{\phi}\left(C_0(\lambda,\tau) - C_1(\lambda,\tau)\right) && \text{(A.1)} \\ &= \lambda n + (1-\lambda)\frac{n^2}{2} - n\sqrt{\lambda\left(1 + (1-\lambda)w_p\right)} + \frac{\lambda}{2}(1-\lambda)w_p. && \text{(A.2)} \end{aligned}$$

By definition, $\lambda_M(n)$ is the minimal λ that satisfies

$$H(\lambda_M(n), n) \leq 0. \tag{A.3}$$

Because $H(0,n) = \frac{n^2}{2} > 0$ and $H(\lambda, n)$ is continuous in both arguments, $\lambda_M(n)$, if it exists, satisfies (A.3) with equality. The condition $H(\lambda, n) = 0$ is a quadratic equation with respect to n, with the following two solutions: $n_1(\lambda) = \frac{w_p\sqrt{\lambda}(1-\sqrt{\lambda})}{\sqrt{1+(1-\lambda)w_p}-1}$ and $n_2(\lambda)$ defined by the right-hand side of (30). The constraint $n_i(\lambda) \geq 1$ is met when $\lambda \geq \frac{1}{w_p}$ for both $i = \{1, 2\}$; moreover, $n_1\left(\frac{1}{w_p}\right) = n_2\left(\frac{1}{w_p}\right) = 1$, and

$$n_2(\lambda) > n_1(\lambda) \geq 1, \forall \lambda \in \left(\frac{1}{w_p}, 1\right]. \tag{A.4}$$

[9] To prove the inequality, see (22) and the discussion following it.

From above analysis, it follows that $\lambda_M(1) = \frac{1}{w_p}$, because it is the smaller of the two values of λ that satisfy $H(\lambda, 1) = 0$ (the other one being $\lambda = 1$). For $n > 1$, we have either $n_1(\lambda_M(n)) = n$ or $n_2(\lambda_M(n)) = n$.

Suppose for some $n \in (1, w_p]$, $n_1(\lambda_M(n))$ exists and is equal to n. Then, by (A.4), we have that $n_2(\lambda_M(n)) > n$. Then, because $n_2(\cdot)$ is strictly increasing from 1 to w_p (cf.(30)), there exists $\lambda' < \lambda_M(n)$ that satisfies $n_2(\lambda') = n_1(\lambda_M(n)) = n$. By definition of $n_2(\cdot)$, we then have that $H(\lambda', n) = 0$, and therefore $\lambda_M(n)$ is not the lowest λ that satisfies $H(\lambda, n) = 0$, contradicting the definition of $\lambda_M(n)$.

Therefore, $\lambda_M(n)$ is defined by identity $n_2(\lambda_M(n)) \equiv n, n \in [1, w_p]$, i.e. $\lambda_M(\cdot)$ is the inverse function of $n_2(\cdot)$. ■